# ENGRI 1101 Introduction to Operations Research

official name: Engineering Applications of Operations Research

Syllabus (draft — subject to change)

Fall 2023



#### **Course description**

This course offers a unique introduction to both mathematical modeling as well as major ideas and algorithms in optimization. The goal of the course is to show that mathematical modeling and algorithms are practical tools, but also to develop a good intuitive understanding of what optimization is, and how to reason about algorithms and optimality.

Classical optimization models will be introduced using applications, with a focus on demonstrating mathematical modeling. Algorithms for these problems will be studied, as well as arguments that optimality of a solution can be verified using arguments that are interpretable and understandable on a basic level.

## **Course objectives**

After this course

- you will be able to formulate a mathematical model for real life problems, using graphs, decision variables, objective functions and constraints;
- you will have a good intuitive understanding of what optimization is;
- you will know algorithms to find optimal solutions to the following optimization problems:
  - the shortest path problem
  - the minimum spanning tree problem
  - the maximum flow problem
  - the transportation problem
  - the assignment problem
  - linear programming problem;
- also you will know approaches for solving integer linear programming problems, in particular you will know what cutting planes are, how to apply the branch-and-bound procedure, and you will have a geometric intuition of these concepts;
- you will be able to apply methods to verify that a solution is optimal, using arguments that are understandable and interpretable on a basic level;
- (*spoiler alert*) you will have an appreciation of the fact that all of the methods in the previous bullet point are intrinsically connected to each other as special cases of duality.

## **Class information**

Lectures: Tuesdays and Thursdays 10.10-11am (Lecture 1) and 12.20-1.10pm (Lecture 2) Dates: August 22nd (first lecture) to November 30th (last lecture) Credit hours: 3 Instruction mode: on campus Expected supplemental hours: approximately 6 hours per week

## **Class format**

Lectures and discussion sections (recitations).

## Class participation and attendance policy

Recitations are mandatory. You are allowed one unexcused absence.

## Prerequisites

This course is self-contained. It assumes no background except high school math.

#### Instructor

Frans Schalekampemailfms9@cornell.eduofficeRhodes 284office hoursWill be announced on Canvas<br/>One-on-one meetings with Frans can be set up at calendly.com/fms9

#### Teaching assistants

To be determined

## Office hours

See Canvas website.

#### **Class websites**

We will be using Canvas to post lecture materials. You should be enrolled automatically into Canvas, but if not, please visit https://canvas.cornell.edu and search for ENGRI 1101.

All announcements for the class will be through Ed Discussion, which is linked from Canvas. We will also use Ed Discussion for questions related to homework, exams and the course content.

Homework will be collected and graded via Gradescope. Regrade requests can also be submitted via Gradescope *within a week of the work being returned*, with a short explanation of the error. Regrade requests without any explanation of why you think there may be a mistake in the grading, will be denied out of hand.

#### Textbook

There is a coursepack for this course, available in the Campus Store. The pdf of the coursepack is available on Canvas, and I therefore made the printed version optional. I do recommend buying it, and downloading the coursepack as well — I always have students who come back to me after a few semesters, asking for the coursepack!

#### Homework

Weekly homework will be due on Thursdays at 11.59pm (Eastern Time) on gradescope.com.

The goal of homework is to make you think deeper about and work with the material in the course, thereby reinforcing the material. If you do not immediately see how to approach a problem, do not ask for help straight away; you learn much more if you manage to find a way forward on your own after a bit of struggle. In other words, the goal of the homework is *not* getting a perfect score, but it is meant is meant as an exercise to sharpen your understanding and to find possible holes in your understanding. If you do not do your own homework, then this setup does

not work. We grade the homework to give you feedback and help you pinpoint which parts of the course you fully understand and which you have to review.

To encourage you to think about the exercises and find solutions yourself, we will scale each homework grade by a factor 10/9 (rounding down to 100 if your scaled score is higher than that). (So getting any score 90% or higher on a homework will be considered a perfect score.)

#### Homework policies

- Grades for each homework are 10/9 times the raw grade for the homework (rounded down to 100 if necessary).
- The lowest assigment grade is dropped.
- No late assignments.
- Regrade requests for homework assignments and exams should be submitted on Gradescope, within a week of when grades are released. We will regrade the entire exam or problem set; your score may go up or down.
- You are allowed and even encouraged to discuss the homework problems with your classmates. However, you *have to write down the solutions on your own*. Don't take notes while discussing the problems with classmates or TAs. In particular, showing each other written solutions is not acceptable.
- Note that TAs do not have the solutions, and may make mistakes. Office hours are not for checking your work for mistakes (this is what grading is for).
- You are not allowed to use sites like Chegg, Coursehero, or other sites that have homework solutions. Posting questions from this course to these sites is a violation of copyright and of Cornell's academic integrity code. Getting answers from these sites deprives you of the chance to struggle with the problems yourself and you really do not want to postpone that struggle to the exam!

(The only way to learn is to practice. This is true for many things in life, and mathematics is no exception. Imagine you wanting to learn to play an instrument, and trying to do so by watching videos on Youtube, instead of practicing the instrument itself — I hope you agree this would be pointless. You can only practice the material in this course if you think about the problems yourself.)

• Violations of the homework collaboration policy or accessing homework solutions found online result in a score of *negative* 50% (-50%) for the assignment.

#### Exams

We will have two midterms and a final during the semester. You will be notified well in advance about their timings and the material of the midterm exams.

#### **Exam policies**

- Clear guidelines will be given about what course materials you will be allowed to access while taking the exams.
- Communicating with any person or accessing unauthorized resources during the exam is considered to be a violation of academic integrity. Not reporting requests of other students who ask for help is also a violation.
- Violations of academic integrity during a prelim or final exam will result in an F in the course.

## Grading

Course grade will be based on homework sets, two tests, one final exam, and recitation exercises.

Homework	20-30%
Test 1	15-20%
Test 2	15-20%
Final Exam	20-40%
<b>Recitation Exercises</b>	10%
Total	100%

I will determine your final grade based on your final score. Your final score guarantees a **minimum** grade as follows: 94% guarantees an A, 82% guarantees a B, 72% a C and 60% a D. I may be more generous than this.

#### Learning outcomes

Here is a list of skills you will have after taking this course. Note that this list is not meant to be exhaustive, and that there may be topics that may not be covered this particular semester. Also, it uses terminology that will be developed during the course, and is therefore meant to be useful at the end of the semester.

After taking this course you will be able to

- define and differentiate between input, constraints, and objectives
- articulate an algorithmic procedure and differentiate between heuristics and optimization
- recognize the iterative nature of modeling
- recognize the power of optimization software
- demonstrate understanding of what it means to learn a problem model and being able to apply it to new settings
- demonstrate understanding of the traveling salesman problem model
- demonstrate understanding of the shortest path model and underlying graph notation

- apply Dijkstra's algorithm to find shortest paths in graphs
- use the node-enticement procedure to verify the output of Dijkstra's algorithm
- explain the logic behind the node-enticements procedure (how it can provide a problem where the solution is obvious, and this solution is also optimal when there are no node-enticements)
- use the shortest path model to solve real-world problems
- demonstrate understanding of the minimum spanning tree model
- apply Prim's algorithm and Kruskal's algorithm to find minimum spanning trees
- use the minimum spanning tree model to cluster high-dimensional data and to compress genomic data
- explain how moats can be used to provide a lower bound on the total length of any spanning tree in general, and how they can prove optimality of a minimum spanning tree in particular
- demonstrate understanding of the maximum flow model and express it using mathematical notation
- apply the Ford-Fulkerson algorithm to find maximum flows
- explain the logic of using residual graphs in the Ford-Fulkerson algorithm
- explain how the capacity of any  $\boldsymbol{s}, t\text{-cut}$  provides an upper bound on the value of any flow
- explain how Dijkstra's algorithm simplifies to the labeling algorithm to find the reachable set from  $\boldsymbol{s}$
- use a minimum cut to verify the optimality of a maximum flow
- use the maximum flow model to find a maximum-cardinality matching in a bipartite graph
- apply bipartite matching to find a request-available car pairing with maximum ETA limited in a ride-share app
- use the maximum flow model and Ford-Fulkerson algorithm to determine whether or not a Baseball team can still end the season in first place
- use a minimum cut to provide clear, non-technical argument that an eliminated team has indeed been eliminated
- demonstrate understanding of the assignment problem and transportation problem models
- use the Hungarian algorithm to find an optimal assignment to the assignment problem
- use the assignment problem to model minimizing the average ETA for a request-available car pairing in a ride-share app
- use node prices to verify an optimal solution to balanced transportation problems
- argue that certain inputs have equivalent solutions, and argue why some are easy to solve
- define and model real-world problems as linear programs
- solve 2-dimensional linear programs using the graphical approach
- use the simplex method to solve general linear programs
- demonstrate understanding of the geometric interpretation of the simplex method
- define and model real-world problems as integer programs
- write out a mathematical model for the Traveling Salesman Problem as an integer program
- explain the relationships between linear and integer programs
- use branch and bound to find optimal solutions to integer programs

- explain the rationale for fathoming nodes in the branch and bound tree
- explain the notion of a cutting plane and their impact in finding optimal solutions to integer programs
- derive the duality of a linear program and interpret it in terms of bounding the original linear program
- explain strong duality
- provide an economic interpretation of dual variables for the Diet Problem linear program.

The learning activities associated with all these outcomes are discussions in lectures and recitations, doing exercises in assignments and exams.

## Academic integrity

Each student in this course is expected to abide by the Cornell University Code of Academic Integrity. Any work submitted by a student in this course for academic credit will be the student's own work. Complete code is available at http://cuinfo.cornell.edu/Academic/AIC.html.

Violations of the Code of Academic Integrity, especially plagiarism, may result in a failing grade in the course.

## Prohibition against buying and selling of course materials

Course materials, posted on Canvas/Piazza or otherwise, are intellectual property belonging to the author/instructor. Students are prohibited against buying or selling any course materials without the express permission of the instructor.

#### Inclusive learning environment

Cornell supports an inclusive learning environment where diversity and individual differences are understood, respected, appreciated, and recognized as a source of strength. It is expected that students in this class will respect differences and demonstrate diligence in understanding how other peoples' perspectives, behaviors, and worldviews may be different from their own.

## Accomodations for students with disabilities

Cornell University is committed to full inclusion for all persons to its educational program and services. Services and reasonable accommodations are available to persons with temporary and permanent disabilities when conditions cause barriers to equal educational opportunity. Student Disability Services (SDS) determines the eligibility of students to receive accommodations and works collaboratively with the student and university faculty and staff to recommend appropriate accommodations. Students are advised to contact SDS as early as possible in the semester to ensure appropriate accommodations: www.sds.cornell.edu, 607-254-4545.

## Attestation

By registering for this class and accessing course materials through Canvas, students agree to abide by University, College, Department, and Course policies.

## **Course schedule**

#### Lectures

This schedule is subject to change as needed during the semester. Changes will be announced and posted to Canvas.

week	day	date	topic	
1	Tuesday	8/22	Introduction: the astronomer's problem	
1	Thursday	8/24	Traveling salesman problem, heuristics	
2	Tuesday	8/29	Reductions	
2	Thursday	8/31	Shortest path Problem 1	
3	Tuesday	9/5	Shortest path Problem 2	
3	Thursday	9/7	Shortest path Problem 3	
4	Tuesday	9/12	Minimum spanning tree problem 1	
4	Thursday	9/14	Minimum spanning tree problem 2	
5	Tuesday	9/19	Maximum flow problem 1	
5	Thursday	9/21	Maximum flow problem 2	
6	Tuesday	9/26	Maximum flow problem 3	
6	Thursday	9/28	Bipartite matching	
7	Tuesday	10/3	Baseball elimination 1	
7	Thursday	10/5	Baseball elimination 2	
8	Tuesday	10/10	break	
8	Thursday	10/12	Transportation problem	
9	Tuesday	10/17	Assignment problem	
9	Thursday	10/19	Recap	
10	Tuesday	10/24	Linear programming 1	
10	Thursday	10/26	Linear programming 2	
11	Tuesday	10/31	Linear programming 3	
11	Thursday	11/2	Geometry of linear programming and the simplex method	
12	Tuesday	11/7	Subtour elimination (I)LP	
12	Thursday	11/9	Cutting planes, branch and bound	
13	Tuesday	11/14	Branch and bound 2	
13	Thursday	11/16	Duality 1	
14	Tuesday	11/21	Duality 2	
14	Thursday	11/23	break	
15	Tuesday	11/28	Recap	
15	Thursday	11/30	Recap	

#### Assignments

This schedule is subject to change as needed during the semester. There are usually 9-10 assignments in this course — the schedule below has more potential due dates than that. Changes will be announced and posted to Canvas.

week	day	date	
2	Thursday	8/31	Assignment due
3	Thursday	9/7	Assignment due
4	Thursday	9/14	Assignment due
5	Thursday	9/21	Assignment due
6	Thursday	9/28	Assignment due
7	Thursday	10/5	Assignment due
8	Thursday	10/12	break
9	Thursday	10/19	Assignment due
10	Thursday	10/26	Assignment due
11	Thursday	11/2	Assignment due
12	Thursday	11/9	Assignment due
13	Thursday	11/16	Assignment due
14	Thursday	11/23	break
15	Thursday	11/30	Assignment due